FORMULAIRE

MODÈLES 1D

Modèle linéaire

$$\frac{\partial u}{\partial t} + \alpha_1 \frac{\partial u}{\partial x} + \alpha_3 \frac{\partial^3 u}{\partial x^3} = \mu_0\, u + \mu_2 \frac{\partial^2 u}{\partial x^2} + \mu_4 \frac{\partial^4 u}{\partial x^4}\,.$$

$$\sigma = \Sigma(k_1) = \mu_0 - \mu_2\, k_1^2 + \mu_4\, k_1^4 \qquad \text{et} \qquad \omega = \Omega(k_1) = \alpha_1\, k_1 - \alpha_3\, k_1^3$$

Linéarisation autour d'un équilibre

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \mu\, u - \gamma\, u^3 + \mu_2 \frac{\partial^2 u}{\partial x^2} + \mu_4 \frac{\partial^4 u}{\partial x^4}$$

$$\frac{\partial \widetilde{u}}{\partial t} + u_0 \frac{\partial \widetilde{u}}{\partial x} = \mu_0\, \widetilde{u} + \mu_2 \frac{\partial^2 \widetilde{u}}{\partial x^2} + \mu_4 \frac{\partial^4 \widetilde{u}}{\partial x^4}$$

Calcul de stabité

$$\frac{\partial \underline{U}}{\partial t} = \underline{\mathcal{F}}(\underline{U}) \qquad \Longrightarrow \qquad \frac{\partial \underline{\widetilde{U}}}{\partial t} = \underline{\underline{L}}\left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)\, \underline{\widetilde{U}}$$

$$\underline{\widetilde{U}}(\underline{x}, t) = \underline{A}\, e^{i\, \underline{k}\cdot\underline{x} + s\, t} \qquad \Longrightarrow \qquad s\, \underline{A} = \underline{\underline{L}}(ik_1, ik_2, ik_3)\, \underline{A}$$

$$\Longrightarrow \qquad \sigma_i = \Sigma_i(\underline{k}) \qquad \text{et} \qquad \omega_i = \Omega_i(\underline{k}) \qquad \text{pour } i = 1, ..., n$$

INSTABILITÉ “ROLL WAVES”

Modèle de Saint Venant

$$\frac{\partial U}{\partial t} + U\,\frac{\partial U}{\partial x} = -g'\,\frac{\partial h}{\partial x} - g\,I - \frac{1}{2}\,C_f\,\frac{U\,|U|}{h} \quad \text{et} \quad \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}\,(U\,h) = 0$$

Linéarisation autour de (h_n, U_n) :

$$\begin{aligned} \frac{\partial \widetilde{U}}{\partial t} + U_n\,\frac{\partial \widetilde{U}}{\partial x} &= -g'\,\frac{\partial \widetilde{h}}{\partial x} + C_f\,\left(\frac{U_n}{h_n}\,\widetilde{U} - \frac{U_n^2}{2\,h_n^2}\widetilde{h}\right) \\ \frac{\partial \widetilde{h}}{\partial t} + U_n\,\frac{\partial \widetilde{h}}{\partial x} + h_n\,\frac{\partial \widetilde{U}}{\partial x} &= 0 \end{aligned}$$

Équations adimensionnées

$$\frac{\partial U_*}{\partial t} + U_*\,\frac{\partial U_*}{\partial x} = -\frac{1}{F^2}\,\frac{\partial h_*}{\partial x} + \frac{1}{2}\,C_f\left(1 - \frac{U_*^2}{h_*}\right) \quad \text{et} \quad \frac{\partial h_*}{\partial t} + \frac{\partial}{\partial x}\,(U_*\,h_*) = 0\ ,$$

Calcul de stabité

$$(S + i\,K + 1)(S + i\,K) + \frac{1}{F^2}\,K^2 + \frac{1}{2}\,i\,K = 0$$

Instable pour tout K dès que $F > 2$.

INSTABILITÉ DE KELVIN HELMOLTZ

Équations d'Euler avec interface

$$\text{div}\ \underline{U}_i = 0 \quad \text{et} \quad \rho_i \frac{d}{dt}\underline{U}_i = -\underline{\text{grad}}\ p_i - \rho_i\ g\ \underline{e}_z \qquad \text{pour } i = 1, 2$$

Conditions aux limites à l'interface

$$\frac{\partial \eta}{\partial t} + \underline{U}_1 \cdot \underline{\text{grad}}\ \eta = w_1\,, \qquad p_1 = p_2\,, \qquad \frac{\partial \eta}{\partial t} + \underline{U}_2 \cdot \underline{\text{grad}}\ \eta = w_2$$

Linéarisation autour de l'équilibre

$$\underline{U}_i = U_i\ x + \underline{\text{grad}}\ \phi_i \qquad \text{et} \qquad \Delta\phi_i = 0$$

avec les conditions aux limites

$$\rho_1 \left[\left(\frac{\partial}{\partial t} + U_1 \frac{\partial}{\partial x}\right)\phi_1 + g\,\eta\right] = \rho_2 \left[\left(\frac{\partial}{\partial t} + U_2 \frac{\partial}{\partial x}\right)\phi_2 + g\,\eta\right] \qquad \text{en } z = 0\ .$$

Calcul de stabité

$$\rho_1 \left[g\,k + (s + i\,k_1\,U_1)^2\right] = \rho_2 \left[g\,k - (s + i\,k_1\,U_2)^2\right]\ .$$

Cas $\rho_1 = \rho_2$: $(s + i\,k_1\,U_1)^2 + (s + i\,k_1\,U_2)^2 = 0$

Cas $U_1 = U_2 = 0$: $s^2 = \frac{\rho_2 - \rho_1}{\rho_1 + \rho_2}\ g\,k$

STABILITÉ DES ÉCOULEMENTS PARALLÈLES

Équation d'Orr-Sommerfeld

$$\left[s + i\,k_1\,U_0(z) - \nu\,(D^2 - k_1^2)\right](D^2 - k_1^2)\;w_m = U_0''(z)\;w_m\;.$$

Théorème du point d'inflexion

Une condition nécessaire pour que $U_0(z)$ soit instable est l'existence d'un point d'inflexion.

Instabilités visqueuses