FORMULAIRE

ONDES SONORES

Modèle linéaire

$$\frac{\partial \tilde{\rho}}{\partial t} = -\rho_0 \,\text{div}\, \underline{U} \;, \quad \rho_0 \,\frac{\partial \underline{U}}{\partial t} = -\text{\underline{grad}}\, \tilde{p} \;, \quad s = s_0 \;, \quad \tilde{p} = c^2 \,\tilde{\rho}$$

avec $(\rho, p, \underline{U}) = (\rho_0, p_0, \underline{0}) + (\tilde{\rho}, \tilde{p}, \text{\underline{grad}}\, \phi)$ et $c^2 = \left(\frac{\partial \mathcal{P}}{\partial \rho}\right)_s (\rho_0, s_0)$

Conservation de l'énergie

$$\frac{\partial W}{\partial t} + \text{div}\, (\underline{I}) = 0 \quad \text{avec} \quad W = \frac{1}{2}\rho_0 \left(\frac{1}{c^2 \rho_0^2}\tilde{p}^2 + \underline{U}^2\right) \quad \text{et} \quad \underline{I} = \tilde{p}\, \underline{U} \;.$$

Relation de dispersion : $\omega = c\, k, \quad \underline{c}_\varphi = \underline{c}_g = c\, \underline{e}_k \;.$

Cas d'une onde monochromatique

$$\langle W_{\text{cin}} \rangle^T = \langle W_{\text{pot}} \rangle^T \;, \quad \langle W \rangle^T = \frac{1}{2}\rho_0\, k^2\, |\phi_m|^2 \quad \text{et} \quad \langle \underline{I} \rangle^T = \underline{c}_g\, \langle W \rangle^T \;.$$

ONDES DE GRAVITÉ INTERNES

Modèle linéaire

$$\text{div}\,\underline{U} = 0, \qquad \frac{d\tilde{\rho}}{dt} = \frac{N^2}{g}\rho_r\, w \quad \text{et} \quad \rho_r\,\frac{d\underline{U}}{dt} = -\underline{\text{grad}}\;\tilde{p} - \tilde{\rho}\; g\,\underline{e}^{(3)}$$

avec $(\rho, p, \underline{U}) = [\rho_0(z), p_0(z), 0] + (\tilde{\rho}, \tilde{p}, \underline{U})$ et $N = \sqrt{-\frac{g}{\rho_r}\frac{d\rho_0}{dz}}$

Conservation de l'énergie

$$\frac{\partial W}{\partial t} + \text{div}\,(\underline{I}) = 0 \quad \text{avec} \quad W = \frac{1}{2}\rho_r\left(\frac{g^2}{\rho_r^2 N^2}\tilde{\rho}^2 + \underline{U}^2\right) \quad \text{et} \quad \underline{I} = \tilde{p}\,\underline{U}\;.$$

Relation de dispersion

$$\omega = N\,|\cos\theta|\;, \quad \underline{c}_\varphi = \frac{\omega}{k}\underline{e}_k \quad \text{et} \quad \underline{c}_g = c_\varphi\;\text{tg}\,\theta\;\underline{e}_\theta \quad \text{avec} \quad \underline{c}_\varphi \cdot \underline{c}_g = 0$$

Cas d'une onde monochromatique

$$\langle W_{\text{cin}}\rangle^T = \langle W_{\text{pot}}\rangle^T\;, \quad \langle W\rangle^T = \frac{\rho_r}{2}\;|w_m|^2\;k^2/k_H^2 \quad \text{et} \quad \langle \underline{I}\rangle^T = \underline{c}_g\;\langle W\rangle^T\;.$$

ONDES DE SURFACE

Modèle linéaire

$$\Delta\phi = 0 \qquad \text{et} \qquad \tilde{p} = -\rho_0 \, \frac{\partial \phi}{\partial t}$$

avec $\dfrac{\partial \phi}{\partial z} = 0$ en $z = -h$ et $\left[\dfrac{\partial \eta}{\partial t} = \dfrac{\partial \phi}{\partial z} \; , \quad \dfrac{\partial \phi}{\partial t} = -g\,\eta\right]$ en $z = 0$.

avec $(\eta, p, \underline{U}) = (0, p_a - \rho_0 g z, \underline{0}) + (\eta, \tilde{p}, \mathrm{g}\underline{\mathrm{ra}}\mathrm{d}\ \phi)$

Conservation de l'énergie

$$\frac{\partial W}{\partial t} + \mathrm{div}\ (\underline{I}) = 0$$

avec $\displaystyle W = \int_{-h}^{\eta} \left(\frac{1}{2}\rho_0\, \underline{U}^2\right)\, dz + \frac{1}{2}\,\rho_0\, g\, \eta^2$ et $\displaystyle \underline{I} = \int_{-h}^{\eta} \tilde{p}\, \underline{U}_H \, dz$

Relation de dispersion

$$\omega = \sqrt{g\; k\;\; \tanh(k\,h)}, \quad \underline{c}_\varphi = \frac{\omega}{k}\underline{e}_k \qquad \text{et} \qquad \underline{c}_g = \underline{c}_\varphi \left(\frac{1}{2} + \frac{k\,h}{\sinh(2\,k\,h)}\right) \; .$$

Cas d'une onde monochromatique

$$\langle W_{\mathrm{cin}}\rangle^T = \langle W_{\mathrm{pot}}\rangle^T \;\; , \quad \langle W\rangle^T = \frac{1}{2}\rho_0\; g\, |\eta_m|^2 \qquad \text{et} \qquad \langle \underline{I}\rangle^T = \underline{c}_g\;\, \langle W\rangle^T \;\; .$$