FORMULAIRE

MODÈLE 2D TURBULENT

Équations de Navier-Stokes incompressibles 2D :

$$\text{div}\, \underline{U} = 0 \qquad \text{et} \qquad \rho_0 \frac{d\underline{U}}{dt} = -\underline{\text{grad}}\, p + \rho_0\, \underline{g} + \rho_0\, \nu_n\, \Delta \underline{U}$$

Conditions aux limites

$$\frac{\partial h}{\partial t} + u\, \frac{\partial h}{\partial x} = w \qquad \text{et} \qquad \underline{\underline{\sigma}}\; \underline{n} = -p_a\, \underline{n} \qquad \text{pour} \qquad z = h(x,t)\ .$$

$$u = w = 0 \qquad \text{pour} \qquad z = 0\ .$$

avec $\underline{\underline{\sigma}} = -p\, \underline{\underline{I}} + 2\, \rho_0\, \nu_n\, \underline{\underline{D}}$

Viscosité turbulente

Viscosité moléculaire ν_n remplacée par $\nu_e = \nu_n + \nu_t$

APPROXIMATION DE MILIEU PEU PROFOND

Équations adimensionnées : α, $\epsilon = \frac{h_0}{L_0}$, $F_r = \frac{U_0}{\sqrt{g\,\cos\alpha\, h_0}}$, $R_e = \frac{h_0\,U_0}{\nu_e}$

$$\begin{aligned}
\frac{\partial u^*}{\partial x^*} + \frac{\partial w^*}{\partial z^*} &= 0 \\
\epsilon\left(\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + w^*\frac{\partial u^*}{\partial z^*}\right) &= -\epsilon\,\frac{\partial p^*}{\partial x^*} + \frac{\mathrm{tg}\,\alpha}{F_r^2} + \frac{1}{R_e}\Delta^* u^* \\
\epsilon^2\left(\frac{\partial w^*}{\partial t^*} + u^*\frac{\partial w^*}{\partial x^*} + w^*\frac{\partial w^*}{\partial z^*}\right) &= -\frac{\partial p^*}{\partial z^*} - \frac{1}{F_r^2} + \frac{\epsilon}{R_e}\Delta^* w^* \\
\text{avec} \qquad \Delta^* &= \epsilon^2\,\frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial z^{*2}}\ .
\end{aligned}$$

Approximation pour $\epsilon \to 0$**,** $\frac{1}{R_e} = O(\epsilon)$ **et** $\mathrm{tg}\,\alpha = O(\epsilon)$

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} &= 0 \\
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial w}{\partial z} &= -g'\,\frac{\partial h}{\partial x} + g\sin\alpha + \nu_e\,\frac{\partial^2 u}{\partial z^2}
\end{aligned}$$

avec les conditions aux limites

$$\begin{aligned}
\frac{\partial u}{\partial z} = 0 \quad \text{et} \quad \frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} = w \quad & \text{pour} \quad z = h(x,t) \\
u = w = 0 \quad & \text{pour} \quad z = 0 \qquad .
\end{aligned}$$

ÉQUATIONS DE SAINT-VENANT

Parmétrisation utilisées

$$\int_0^h \widehat{u}^2 \, dz = \beta \, U^2 \, h \qquad \text{et} \qquad \tau_f = \frac{1}{2} \, C_f \, \rho_0 \, U \, |U|$$

On choisit $\beta = 0$ et C_f constant ou sous la forme $C_f(h, U)$.

$$\text{Manning-Strickler :} \quad C_f(h) = \frac{2\, g}{K^2 \, h^{1/3}} = 0,1 \, \left(\frac{z_0}{h}\right)^{1/3}$$

Équations de Saint-Venant pour $\epsilon \to 0$, $\frac{1}{R_e} = O(\epsilon)$ et $\mathrm{tg}\,\alpha = O(\epsilon)$:

$$\begin{aligned} \frac{\partial h}{\partial t} + U \, \frac{\partial h}{\partial x} + h \, \frac{\partial U}{\partial x} &= 0 \\ \frac{\partial U}{\partial t} + U \, \frac{\partial U}{\partial x} + g' \, \frac{\partial h}{\partial x} &= g \, \sin\alpha - \frac{C_f}{2} \, \frac{U|U|}{h} \, . \end{aligned}$$

Approximation des intumescences : $\frac{1}{R_e} \ll \epsilon$ et $\mathrm{tg}\,\alpha \ll \epsilon$

$$\frac{\partial h}{\partial t} + U \, \frac{\partial h}{\partial x} + h \, \frac{\partial U}{\partial x} = 0 \qquad \text{et} \qquad \frac{\partial U}{\partial t} + U \, \frac{\partial U}{\partial x} + g' \, \frac{\partial h}{\partial x} = 0 \, .$$

Approximation des ondes de crues : $\epsilon \to 0$, $\frac{1}{R_e} = O(1)$ et $\mathrm{tg}\,\alpha = O(1)$

$$\frac{\partial h}{\partial t} + U \, \frac{\partial h}{\partial x} + h \, \frac{\partial U}{\partial x} = 0 \quad \text{avec} \quad g \sin\alpha = \frac{C_f}{2} \, \frac{U|U|}{h} \, .$$