FORMULAIRE

CLASSIFICATION DES EDP 1D

Forme standard

$$\underline{\underline{A}}\,\partial_t\,\underline{U} + \underline{\underline{B}}\,\partial_x\,\underline{U} = \underline{\underline{D}}\,\underline{U}$$

Classification par relation de dispersion

$$\det\left(s\,\underline{\underline{A}} + i\,k\,\underline{\underline{B}} - \underline{\underline{D}}\right) = 0\ .$$

$$s = S_n(k) = \Sigma_n(k) - i\,\Omega_n(k) \quad \text{avec} \quad n = 1, ..., M$$

Classification par existence de caractéristiques

$$\det\left(-\lambda\,\underline{\underline{A}} + \underline{\underline{B}}\right) = 0\ .$$

$$(\partial_t + \lambda_n\,\partial_x)\;J_n = L_n(J_1, ..., J_N) \quad \text{pour} \quad n = 1, ..., N\ .$$

ONDES DE SURFACE HYPERBOLIQUES

Équations de Saint Venant linéaire

$$\begin{cases} \partial_t\,\widetilde{h} + U_0\,\partial_x\,\widetilde{h} + h_0\,\partial_x\,\widetilde{U} &= 0 \\ \partial_t\widetilde{U} + g\,\partial_x\,\widetilde{h} + U_0\,\partial_x\,\widetilde{U} &= 0 \end{cases}$$

Modes propres

$$\Omega_{1,2}(k) = \left(U_0 \pm \sqrt{g\,h_0}\right)\,k \qquad \text{et} \qquad \underline{\phi}_{1,2} = \begin{pmatrix} \pm\sqrt{\frac{h_0}{g}} \\ 1 \end{pmatrix}$$

Invariants de Riemann

$$\partial_t\,J_{1,2} + \lambda_{1,2}\,\partial_x\,J_{1,2} = 0$$

$$\lambda_{1,2} = U_0 \pm \sqrt{g\,h_0} \qquad \text{et} \qquad J_{1,2} = \widetilde{U} \pm \sqrt{\frac{g}{h_0}}\,\widetilde{h}$$

ONDES DE SURFACE DISPERSIVES

Équation d'Euler irrotationnelles linéarisées

$$\begin{cases} \partial_t \widetilde{h} - \partial_z \widetilde{\phi} = 0 \\ \partial_t \widetilde{\phi} + g\,\widetilde{h} = 0 \end{cases} \text{en } z = h_0 \,, \qquad \Delta\widetilde{\phi} = 0 \qquad \text{et} \quad \partial_z \widetilde{\phi} = 0 \text{ en } z = 0$$

Relations de dispersion

$$\Omega_{12}(k) = \pm\text{sign}(k)\;\Omega_a(k) \quad \text{avec} \quad \Omega_a(k) = \sqrt{g\;k\;\tanh(k\,h_0)}$$

Vitesse de groupe

$$c_g(k) = c_\varphi(k)\left[\frac{1}{2} + \frac{k\,h_0}{\sinh(2\,k\,h_0)}\right]\;.$$